PER-UNIT SYSTEMS FOR ELECTRIC MACHINES

Introduction

A per-unit quantity Q is defined as the ratio of an actual quantity Q_A to an appropriately chosen base value Q_N so that

$$Q = Q/Q$$

The notation here is to use the color red for per-unit values, blue for actual values, and magenta for normalizing values. If Q and Q are of the same order of magnitude, Q takes on a value close to unity. So while Q and Q vary widely in absolute values depending on the size or rating of the device to which they relate and have dimensions such as Volt, Ampere, Ohm, Weber, Newton-meter, their quotient Q is a dimensionless relative quantity that lies in a narrow numerical range. In the context of electric machines, the following advantages to the use of per-unit (pu) parameters and pu equations can be cited:

- Per-unit parameters do not vary appreciably over a wide range of machine ratings, thereby providing a better characterization and comparison of differing designs.
- A pu system simplifies the analysis of machine systems because it eliminates the consideration of such cumbersome factors as numbers of poles and phases, winding turns ratios; in effect, it allows to view the machine on the basis of an equivalent two-phase, two-pole configuration with coupled windings having 1:1 turns ratios.
- When dealing with an interconnected system of machines of different ratings, a common overall system normalization provides an effective way to account for the interconnections.

Machine rating

The rating of an electric machine invariably serves as the basis for determining the normalizing factors required to define the pu system. Typically, the rating (as it normally appears on the nameplate) provides most of the following information:

- frequency = f_N (Hz)
- number of poles = p
- number of phases = m (usually 3)
- speed = n_N (rpm)

- rated voltage = V_r (usually on a line-to-line rms basis) (V)
- power factor = pf_N
- efficiency = η_N
- electrical power output P_e (Watts) for a generator; P_N = P_e / pf_N (VA)
- mechanical power output P_m (hp) for a motor; P_N = 746 x P_m (W)

Base values

- Base frequency $\omega_0 = 2\pi f_N$ [rad/s]
- Base Power = P_N [W]
- Base voltage = peak phase voltage = $V_N = \sqrt{\frac{2}{3}} V_r$ [V]
- Base peak current = $I_N = \dfrac{2P_N}{3V_N}$ [A]
- Base impedance = $Z_N = \dfrac{V_N}{I_N}$ [Ω]
- Base flux linkage = $\lambda_N = \dfrac{V_N}{\omega_0}$
- Base inductance = $L_N = \dfrac{\lambda_N}{I_N} = \dfrac{Z_N}{\omega_0}$
- Base synchronous speed = $\omega_{mecN} = \dfrac{2\omega_0}{p}$
- Base torque = $\dfrac{P_N}{\omega_{mecN}} = \dfrac{pP_N}{2\omega_0}$

Induction machine equations

SI units

$$\overline{v}_s = R_s \overline{i}_s + \frac{d\overline{\lambda}_s}{dt} + \omega_k M \overline{\lambda}_s$$

$$\overline{v}_r = R_r \overline{i}_r + \frac{d\overline{\lambda}_r}{dt} + (\omega_k - \omega_m) M \overline{\lambda}_r$$

$$T_e = \frac{3}{2}\frac{p}{2}\overline{\lambda}_s \otimes \overline{i}_s = \frac{3}{2}\frac{p}{2} L_m \overline{i}_r \otimes \overline{i}_s = J\frac{d\omega_{mecN}}{dt} + T_L$$

Per unit

Dividing the first 2 equations by the base values

$$V_N = Z_N I_N = \omega_0 \lambda_N$$

yields the 2 pu equations

$$\overline{v}_s = R_s \overline{i}_s + \frac{1}{\omega_0}\frac{d\overline{\lambda}_s}{dt} + \omega_k M \overline{\lambda}_s$$

$$\overline{v}_r = R_r \overline{i}_r + \frac{1}{\omega_0}\frac{d\overline{\lambda}_r}{dt} + (\omega_k - \omega_m) M \overline{\lambda}_r$$

Dividing the mechanical equation by the bases

$$T_N = \frac{P_N}{\omega_{mecN}} = \frac{p}{2}\frac{3}{2}\frac{V_N I_N}{\omega_0} = \frac{p}{2}\frac{3}{2}\lambda_N I_N = \frac{p}{2}\frac{3}{2} L_N I_N I_N$$

establishes the pu mechanical equation

$$T_e = \overline{\lambda}_s \otimes \overline{i}_s = L_m \overline{i}_r \otimes \overline{i}_s = 2H\frac{d\omega_m}{dt} + T_L$$

where the so-called inertia constant $H = \frac{1}{2}\frac{J\omega_{mecN}{}^2}{P_N}$ [s]

Flux linkage – current relations

SI	Base	pu
$\overline{\lambda}_s = L_s \overline{i}_s + L_m \overline{i}_r$	$\lambda_N = L_N I_N$	$\overline{\lambda}_s = L_s \overline{i}_s + L_m \overline{i}_r$
$\overline{\lambda}_r = L_m \overline{i}_s + L_r \overline{i}_r$		$\overline{\lambda}_r = L_m \overline{i}_s + L_r \overline{i}_r$

Power relations

$$P = v_A i_A + v_B i_B + v_C i_C = \frac{3}{2}(v_a i_a + v_b i_b) = \frac{3}{2}(v_d i_d + v_q i_q) \quad \text{(SI)}$$

$$P_N = \frac{3}{2} V_N I_N \quad \text{(Base)}$$

$$P = \frac{2}{3}(v_A i_A + v_B i_B + v_C i_C) = v_a i_a + v_b i_b = v_d i_d + v_q i_q \qquad \text{(pu)}$$

Note regarding motor and generator conventions:

The so-called motor convention has been adopted in most of the simulations; thus, rated output horsepower is chosen as basis so that

$$P_N = hp_{rated} \times 746 \text{ W}$$

and, under rated conditions, T = 1, V = 1, but $I = \dfrac{1}{\eta_N (pf_N)} \neq 1$.

For the generator convention, base power is chosen to be equal to the volt-ampere rating of the machine. As a consequence, for rated conditions, the per-unit values are such that V = 1, I = 1, and $P = pf_N$.

However, since $T_{rated} = \dfrac{P_{out,rated}}{\omega_{mecN}} = \eta_N (pf_N) \dfrac{P_N}{\omega_{mecN}}$. $T = \eta_N (pf_N) \neq 1$

If both conventions are to be used, the following expressions relate the per-unit values defined in both systems denoted by the subscripts VA and HP respectively:

$$\frac{T_{VA}}{T_{HP}} = \frac{I_{VA}}{I_{HP}} = \frac{Z_{HP}}{Z_{VA}} = \eta_N (pf_N)$$

SI or pu formulation?

An immediate way to ascertain if the machine equations or their Simulink block representations are expressed in a per-unit form is to examine the time differential operator **dt** in which **t** is always expressed in seconds: when **dt** or a corresponding integrator block is multiplied by the base frequency ω_o, the per-unit formulation is being used.

Example:

A three-phase Y-connected 220-V (line-to-line) 3-hp four-pole induction motor has the following equivalent-circuit constants in ohms per phase referred to the stator:

R_s = 0.435 R_r = 0.185 X_{sl} = 0.755 X_{rl} = 0.755 X_m = 26.13

The moment of inertia is J = 0.089 Kg-m^2.

The base values are then:

P_N = 3x746 = 2238 W $\qquad V_N = \sqrt{\frac{2}{3}} V_r$ = 179.63 V $\qquad I_N = \frac{2P_N}{3V_N}$ = 8.306 A

$Z_N = \frac{V_N}{I_N}$ = 21.63 Ω $\qquad \omega_{mecN} = \frac{2(2\pi 60)}{4}$ = 188.5 rad/s

The per-unit values of the parameters become

$R_s = R_s / Z_N = 0.020$ $\qquad R_r = R_r / Z_N = 0.0377$ $\qquad X_{sl} = L_{sl} = X_{sl} / Z_N = 0.0349 = L_{rl}$

$L_m = X_m = X_m / Z_N = 1.208$

$$H = \frac{1}{2}\frac{J\omega_{mecN}^{\;\;2}}{P_N} = \frac{1}{2}0.089\frac{188.5^2}{2238} = 0.7065 \text{ s}$$

Table

#	Power [hp]	Poles	V_{LL} [V]	R_s [pu]	L_{sl} [pu]	R_r [pu]	L_{rl} [pu]	L_m [pu]	H [s]
1	3	4	220	0.0201	0.0349	0.0377	0.0349	1.2081	0.7065
2	25	4	460	0.0219	0.0498	0.0472	0.0498	1.9504	0.5277
3	50	4	460	0.0153	0.0532	0.0402	0.0532	2.3061	0.7916
4	100	4	460	0.0109	0.0532	0.0472	0.0532	2.5121	1.0595
5	250	4	2300	0.0241	0.0864	0.0141	0.0864	3.0263	0.6591
6	500	4	2300	0.0185	0.0851	0.0132	0.0851	3.8092	0.5269
7	800	4	2300	0.0148	0.0808	0.0106	0.0808	4.0702	0.6329
8	1000	4	2300	0.0158	0.0851	0.0104	0.0851	7.6343	0.7113
9	1500	4	2300	0.0118	0.0797	0.0078	0.0797	4.2026	0.7072
10	2250	4	2300	0.0092	0.0718	0.0071	0.0718	4.1388	0.6761
11	6000	4	4160	0.0057	0.0781	0.0057	0.0781	5.7431	2.6791

Typical Parameters for 3-phase Y-connected 60-Hz Induction Machines
(in per unit based on hp output)